When dealing with collisions in 2-dimensions it is important to remember that momentum is a vector with magnitude and direction. When finding the total momentum we have to do: vector addition.

### Collisions at 90°:
A 750 kg Peugeot travelling at 21 m/s West collides with a 680 kg Fiat travelling at 18 m/s South. If the two cars become entwined what is their total final velocity?

\[
\begin{align*}
\rho_1 &= m_1 V_1 = (750)(21) = 15750 \text{ kg m/s} \\
\rho_2 &= m_2 V_2 = (680)(21) = 14280 \text{ kg m/s} \\
\rho_T &= \sqrt{\rho_1^2 + \rho_2^2} = \sqrt{15750^2 + 14280^2} = 19947 \text{ kg m/s} \\
\theta &= \tan^{-1}\left(\frac{15750}{14280}\right) = 38^\circ \text{ South} \\
V_f &= \frac{\rho_T}{m_T} = \frac{19947}{14280} = 1.4 \text{ m/s}
\end{align*}
\]

Remember that it is momentum that is conserved, so we need to add the momenta NOT velocities.

### Collisions not at 90° (because life is never that easy…):
A 4.0 kg bowling ball is moving east at an unknown velocity when it collides with a 6.1 kg frozen cantaloupe at rest. After the collision, the bowling ball is traveling at a velocity of 2.8 m/s 32° N of E and the cantaloupe is traveling at a velocity of 1.5 m/s 41° S of E. What was the initial velocity of the bowling ball?

**Before**

\[ V = ? \]

\[ V = 0 \]

**After**

\[ 2.8 \text{ m/s} \]

\[ 32° \text{ N of E} \]

\[ 1.5 \text{ m/s} \]

\[ 41° \text{ S of E} \]

**Component Method**

We need to break the final momenta of the two objects into x and y components:

\[
\begin{align*}
\rho_x &= m_1 V_1 \cos 32° = 9.498 \\
\rho_y &= m_1 V_1 \sin 32° = 5.935 \\
\rho_2 &= m_2 V_2 = 9.15 \text{ kg m/s} \\
\rho_2 x &= \rho_2 \cos 41° = 6.906 \\
\rho_2 y &= \rho_2 \sin 41° = 4.617 \\
\rho_T &= \sqrt{\rho_2 x^2 + \rho_2 y^2} = 6.003
\end{align*}
\]

Remember: when using components to include +/- signs.
We then add the **individual x** and the **individual y** components to find our total momentum.

\[ \Sigma p_x = p_{1x} + p_{2x} = 9.998 + 6.906 = 16.904 \text{ kg m/s} \]

\[ \Sigma p_y = p_{1y} + p_{2y} = 5.935 + (-6.003) = -0.068 \text{ kg m/s} \approx 0 \]

Notice that the total momentum is all in the **x direction**! This should be no surprise since the bowling ball was initially only moving in the x direction.

Don’t forget to solve for the initial velocity (magnitude and direction):

\[
\rho_i = m_i v_{ii} \quad v_{ii} = \frac{\rho_i}{m_i} = \frac{16.404}{9.0} = 1.823 \text{ m/s}
\]

**Vector Addition:**

Simply add the vectors and solve with the sine or cosine law. Notice that the **total momentum** is either the initial or the final because momentum is **conserved**.

First we need to use geometry to solve for the angle opposite the total momentum.

And then, start hammering:

\[
\rho_+ = \rho_1 = \rho_2 = 11.2 \text{ kg m/s}
\]

\[
\rho_+ = \frac{11.2 \sin 107}{\sin 91} = 16.33 \text{ kg m/s}
\]

\[
v_i = \frac{\rho_i}{m_i} = \frac{16.33}{4.0} = 4.1 \text{ m/s}
\]